1. **Proposed by Professor G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.**

Three persons A, B, C, throw with three dice. They each stake \$10.00 and the one who first throws at least ten with the three dice takes the whole stake. Find the expectation of each.

Solution by the Proposer.

The chance of throwing respectively 10, 11, 12, ..., 18, with three dice is $\frac{27}{216}, \frac{27}{216}, \frac{25}{216}, \frac{21}{216}, \frac{15}{216}, \frac{10}{216}, \frac{6}{216}, \frac{3}{216}, \frac{1}{216}$.

The chance of throwing at least ten is equal to the sum of all these chances $=\frac{135}{216}=\frac{5}{8}$.

The chance that A will win $=\frac{5}{8}$, that he will not win $=\frac{3}{8}$. The chance that B will win $=\frac{5}{8}.\frac{3}{8}$, that he will not win $=(\frac{3}{8})^2$. The chance that C will win $=\frac{5}{8}.(\frac{3}{8})^2$. The chance that A will win the second throw $=\frac{5}{8}.(\frac{3}{8})^3$, that B will win $=\frac{5}{8}.(\frac{3}{8})^4$, that C will win $=\frac{5}{8}.(\frac{3}{8})^5$.

$$\begin{aligned}
\therefore\quad A\text{'s chance} &= \tfrac{5}{8}+\tfrac{5}{8}(\tfrac{3}{8})^3+\tfrac{5}{8}(\tfrac{3}{8})^6+\ldots.=\tfrac{64}{97},\\
B\text{'s chance} &= \tfrac{5}{8}(\tfrac{3}{8})+\tfrac{5}{8}(\tfrac{3}{8})^4+\tfrac{5}{8}(\tfrac{3}{8})^7+\ldots.=\tfrac{24}{97},\\
C\text{'s chance} &= \tfrac{5}{8}(\tfrac{3}{8})^2=\tfrac{5}{8}(\tfrac{3}{8})^5+\tfrac{5}{8}(\tfrac{3}{8})^8+\ldots.=\tfrac{9}{97},\\
A\text{'s expectation} &= (\tfrac{64}{97}\times 30)-10=\$9\tfrac{77}{97},\\
B\text{'s expectation} &= (\tfrac{24}{97}\times 30)-10=-\$2\tfrac{56}{97},\\
C\text{'s expectation} &= (\tfrac{9}{97}\times 30)-10=-\$7\tfrac{21}{97}.
\end{aligned}$$

Also solved by *L. V. Roy*, *P. H. Philbrick*, *J. F. W. Scheffer*, *H. C. Whitaker*, and *W. H. Draughon*

PROBLEMS.

4. **Proposed by G. B. M. ZERR, Principal of High School, Staunton, Virginia.**

Four points taken at random in each half, made by the transverse axis, of an ellipse, are joined in such a way by straight lines as to enclose an octagonal surface; find the mean area of this surface.

5. **Proposed by DE VOLSON WOOD, M. A., C. E., Professor of Mechanical Engineering, Stevens Institute of Technology, Hoboken, New Jersey.**

An actual case suggested the following:

An equal number of white and black balls of equal size are thrown into a rectangular box, what is the probability that there will be contiguous contact of white balls from one end of the box to the opposite end? As a special example, suppose there are 30 ball in the length of the box, 10 in the width, and 5 (or 10) layers deep.

Solutions to these problems should be received on or before May 1st.

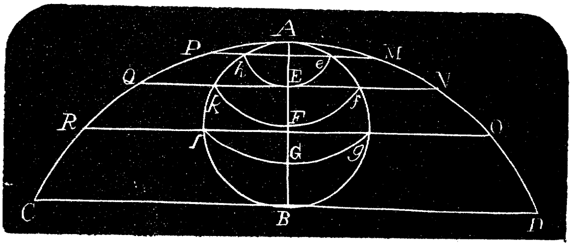
MISCELLANEOUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

1. **Proposed by G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.**

To divide the arc of a cycloid into eight equal parts.

I. Solution by ALFRED HUME, C. E., Professor of Mathematics, University of Mississippi; H. W. DRAUGHON, Clinton, Louisiana; and the Proposer.



Let CAD be the cycloid, AB the diameter of generating circle. Divide AB into four equal parts at E, F, G, and with A as a center and radii equal to AE, AF, AG, draw arcs cutting the circle at e, h; f, k; g, l. Through e, h; f, k; g, l draw parallels to CD. Then, arc $AM=2$ chord $Ae=2AE$,

arc $AN=2$ chord $Af=2AF=4AE$,

arc $AO=2AG$, arc $AD=2AB$.

$\therefore AN=2AM,\ AO=3AM,\ AD=4AM.$

$\therefore AM=MN=NO=OD=AP=PQ=QR=RC.$

II. Solution by WILLIAM SYMMONDS, A. M., Professor of Mathematics and Astronomy, Pacific College, Santa Rosa, California.

Suppose the curve to be divided into n equal parts, n being any integral number. Through the points of division, which are points symmetrical with respect to the axis, let right lines be drawn. Call the distances severally from the points where these lines intersect the axis to the vertex equal to x.

Let $2r$ = height of the cycloid; then $8r$ = the entire length of the curve. If n be odd, $\sqrt{8rx}=\frac{1}{2n}.\,8r,\ \frac{3}{2n}.8r,\ \frac{5}{2n}.8r$, etc.

Whence respectively,

$$(1)\ x=\left(\frac{1}{2n}\right)^2.\,8r,\ \left(\frac{3}{2n}\right)^2.\,8r,\ \left(\frac{5}{2n}\right)^2.8r\ldots\ldots\left(\frac{n-2}{2n}\right)^2.8r=\left(\frac{1}{n}\right)^2 2r,$$
$$\left(\frac{3}{n}\right)^2 2r,\left(\frac{5}{n}\right)^2 2r\ \cdots\cdots\left(\frac{n-2}{2n}\right)^2 2\,r.$$

If n be even, $\sqrt{8rx}=\frac{0}{n},\ \frac{8r}{n},\ \frac{16r}{n},\ \frac{24r}{n}$, etc.

$$\therefore\ (2),x=\left(\frac{0}{n}\right)^2 8r,\ \left(\frac{1}{n}\right)^2 8r,\ \left(\frac{2}{n}\right)^2 8r,\ \left(\frac{3}{n}\right)^2 8r,\ \text{etc.}$$

Making $n=8$, in (2), we have, $x=0,\ \frac{r}{8},\ \frac{r}{2},\ \frac{9}{8}\,r$. If these distances be measured down the axis from the vertex, the ordinates to the points so determined will mark the curve in the points of division required.

Also solved by *P. H. Philbrick*, *H. C. Whitaker*, *C. E. Myers*, and *J. H. Beach*.

2. Proposed by SYLVESTER ROBINS, North Branch Depot, New Jersey.

Give the diminsions of thirteen rational trapezoids each one having 1885 for its parallel bisector; and as many more wherein each bisector is 1105.

Solution by A. H. BELL, Hillsboro, Illinois.

The bisector 1885 of the trapezoid, also the bisector 1105, given in the problem, are composed of the product of three prime numbers, of the form of